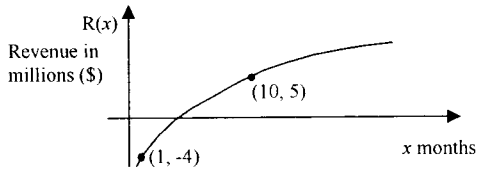
Name : _____

SQUARE ROOT: PROBLEMS

1 After one month in operation, a company's revenue has grown according to the formula $R(x) = a\sqrt{x-h} + k$ where R(x) is the profit in millions of dollars after a period of x months.



Losses amounted to $400 the first month. After ten months, $5000 in profits was recorded.

After how many months in operation did the company begin to make a profit?

Show your work.

2 A real function is defined in the interval [0, 9] by

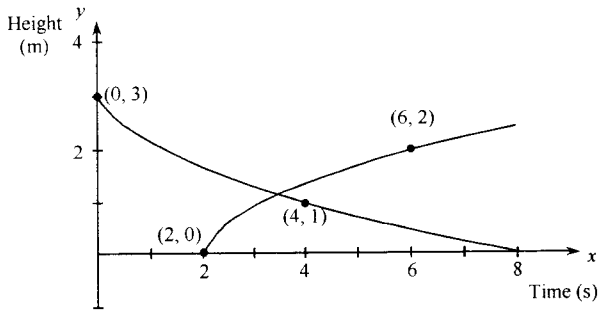
$$f(x) = 2 - \sqrt{9-x}.$$

Draw the graph of this function.

3 What is the solution set of the following inequality?

$$2\sqrt{x-7} + 5 < 0.2x + 8.4$$

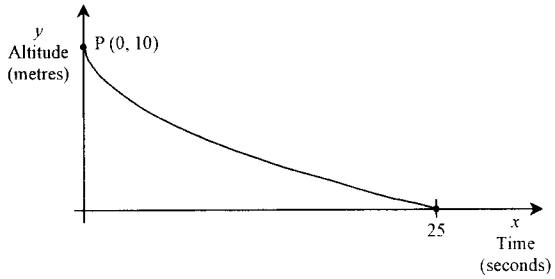
4 Two missiles are launched 2 seconds apart. The paths they follow over a span of 8 seconds can be represented by two different square root functions, as illustrated below:



How many seconds after the 2nd projectile has been launched, will it be higher than the 1st projectile?

Show all your work.

5 Caroline slides down a waterslide at an aquatic park. The following graph represents Caroline's altitude in relation to her sliding time. This curve represents a square root function whose vertex is point P(0, 10).



A section of the slide is covered by a tarpaulin, forming a tunnel. Caroline enters the tunnel when she is at an altitude of 6 m. She exits the tunnel at an altitude of 4 m.

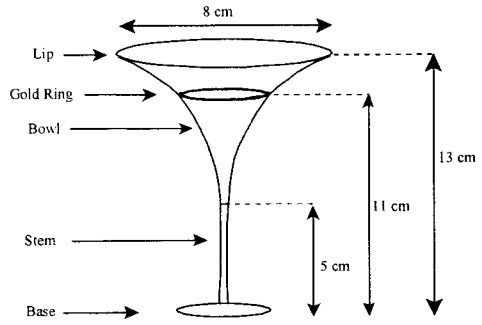
How long was Caroline in the tunnel?

Show all your work.

6 A new glass has been designed by rotating part of a graph of a square root function about the axis containing the stem of the glass. (Assume the width of the stem to be zero.)

As illustrated in the diagram, the diameter of the lip of the bowl is 8 centimetres. The glass stands 13 centimetres in height and the top of the stem of the glass is 5 centimetres high.

A decorative gold ring is to be painted around the bowl 11 centimetres from the bottom of the glass, at a cost of 2 cents per centimetre.



How much will it cost to paint the gold ring around the bowl?

Round your answer to the nearest cent.

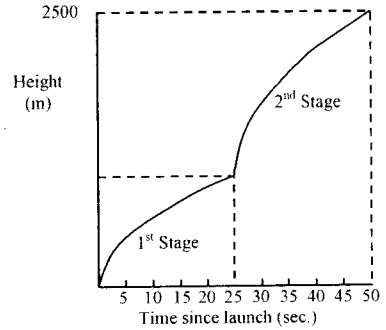
Show all your work.

7 A two-stage model rocket is launched from ground level. Its 1st stage (engine) powers the rocket vertically, according to the rule $H(t) = 200\sqrt{t}$,

where $H(t)$ is height, in metres,
and t is time, in seconds, after launch.

At 25 seconds, the exhausted 1st stage is ejected, and the 2nd stage fires. The height of the rocket after the first 25 seconds can be expressed according to a new square root function of the form $y = a\sqrt{(x - h)} + k$.

50 seconds after the initial launch the rocket reaches a height of 2500 m.

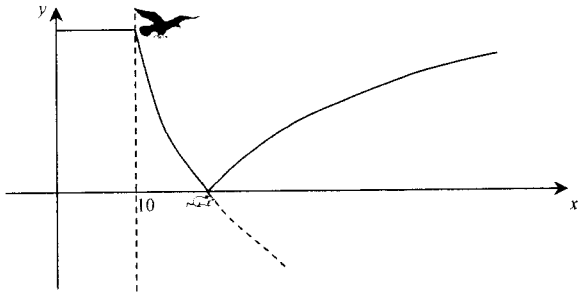


Rounded to the nearest metre, what is the height of the rocket 11 seconds after the firing of the 2nd stage?

8 After flying for 10 seconds, a hawk swoops down, catches a mouse, and immediately takes off with its prey.

The path of the hawk's descent has been determined to be in the shape of a rational function and the path of its ascent is in the shape of a square root function. The point of ascent corresponds to the vertex of the square root function. Four seconds after the hawk catches the mouse, it is 8 m above the ground.

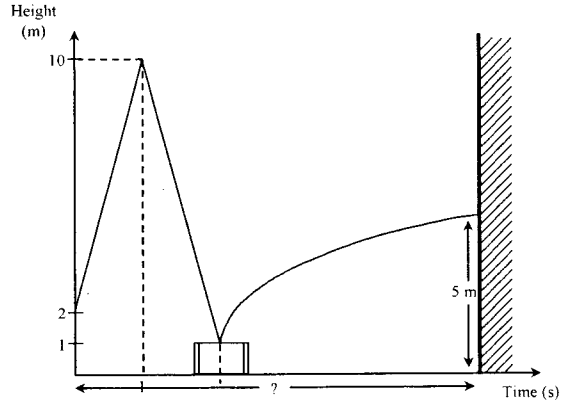
The rational function is $f(x) = \frac{4}{x - 10} - 2$.



How many metres above the ground will the hawk be 9 seconds after it catches its prey?

Show all your work.

9 A tennis ball is hit by a racket from a height of 2 metres and follows the path of an absolute value function. One second later the ball hits the ceiling, which has a height of 10 metres. On its way down, the ball bounces off a table that is 1 metre high. After the bounce, its path is a semi parabola. One second after the ball hits the table, it reaches a height of 3 metres before hitting a wall at a height of 5 metres.



How many seconds after the ball was hit by the racket did it hit the wall?

Show all your work.

5 Example of an appropriate method

Rule of the function

x : time in seconds

$f(x)$: altitude in metres

$f(x) = a\sqrt{x - h} + k$

$f(x) = a\sqrt{x - 0} + 10$

$f(x) = a\sqrt{x} + 10$

$f(25) = 0$ therefore $0 = a\sqrt{25} + 10$

$-10 = 5a$

$-2 = a$

$f(x) = -2\sqrt{x} + 10$

Time at which Caroline entered the tunnel

$f(x) = 6$ therefore $6 = -2\sqrt{x} + 10$

$-4 = -2\sqrt{x}$

$2 = \sqrt{x}$

$4 = x$

After 4 seconds, Caroline entered the tunnel.

Time at which Caroline exited the tunnel

$f(x) = 4$ therefore $4 = -2\sqrt{x} + 10$

$-6 = -2\sqrt{x}$

$3 = \sqrt{x}$

$9 = x$

After 9 seconds, Caroline exited the tunnel.

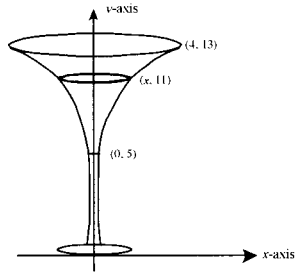
Time during which Caroline was in the tunnel

$9 - 4 = 5$ seconds

Answer: Caroline was in the tunnel for **5** seconds.

Note: Students who used an appropriate method in order to determine the rule of the function have shown that they have a partial understanding of the problem.

6 Example of an appropriate solution



The square root function must be in the form: $y = a\sqrt{x} + k$

Substituting (0, 5) we get: $5 = a\sqrt{0} + k$ So $k = 5$

Substituting (4, 13) we get: $13 = a\sqrt{4} + 5$ So $a = 4$

So the function is: $y = 4\sqrt{x} + 4$

At the ring, $y = 11$ $11 = 4\sqrt{x} + 5$ So $x = 2.25$

But 2.25 represents the radius of the gold ring in centimetres.

So the circumference is: $C = 2\pi (2.25) \approx 14.13$ cm

Therefore the gold ring will cost: $14.13 \times 2 = 28.26$ cents.

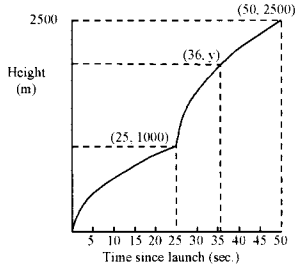
Answer Rounded to the nearest cent, the cost of the gold ring is 28 cents.

Note Do not deduct any marks if the student wrote 29 cents.

7 Example of an appropriate method

Step 1: Find the height of the rocket at 25 seconds

$H(25) = 200\sqrt{25} = 200(5) = 1000$ m



Step 2: Find the equation of the 2nd stage

$H_2(t) = a\sqrt{x - h} + k$

Using $(h, k) = (25, 1000)$, we get

$H_2(t) = a\sqrt{x - 25} + 1000$

Substituting (50, 2500), we get

$2500 = a\sqrt{50 - 25} + 1000$

$2500 = a\sqrt{25} + 1000$

$2500 = 5a + 1000$

$5a = 1500$

$a = 300$

Therefore, $H_2(t) = 300\sqrt{x - 25} + 1000$

11 seconds after the firing of the second stage (at 25 sec.) is 36 sec.

Step 3: Find the image of 36 in H_2

$H_2(36) = 300\sqrt{36 - 25} + 1000 = 300\sqrt{11} + 1000 \approx 1994.99$ m

Answer Rounded to the nearest metre, the height of the rocket 11 seconds after firing of the 2nd stage was 1995 m.

Do not penalize students for not rounding off correctly.

8 Example of an appropriate solution

Find the zero of the rational function

$$f(x) = \frac{4}{x - 10} - 2$$

$$0 = \frac{4}{x - 10} - 2$$

$$2 = \frac{4}{(x - 10)}$$

$2(x - 10) = 4$

$x - 10 = 2$

$x = 12$

Vertex of square root function (12, 0)

$y = a\sqrt{x - h} + k$

$y = a\sqrt{x - 12}$

$8 = a\sqrt{16 - 12}$

$8 = a\sqrt{4}$

$8 = 2a$

$4 = a$

$y = 4\sqrt{x - 12}$

9 seconds later

$y = 4\sqrt{21 - 12}$

$y = 4\sqrt{9}$

$y = 4(3)$

$y = 12$

Answer: Nine seconds after it catches its prey, the hawk will be **12** metres above the ground.

Note: Students who have found the vertex of the square root function have shown they have a partial understanding of the problem.

9 Example of an appropriate solution

Find the equation of the absolute value function

Vertex (1, 10) point (0, 2)

$y = a|x - 1| + 10$

$2 = a|-1| + 10$

$-8 = a$

$y = -8|x - 1| + 10$

Find the x value when $y = 1$

$y = -8|x - 1| + 10$

$1 = -8|x - 1| + 10$

$-9 = -8|x - 1|$

$\frac{9}{8} = |x - 1|$

$\therefore \quad x - 1 = \frac{-9}{8}$ or $x - 1 = \frac{9}{8}$

$x = \frac{-1}{8}$ $\qquad$ $x = \frac{17}{8}$

$x = \frac{17}{8}$ or 2.125

Find the equation of the square root function

Starting point (2.125, 1) point (3.125, 3)

$y = a\sqrt{x - 2.125} + 1$

$3 = a\sqrt{3.125 - 2.125} + 1$

$2 = a\sqrt{1}$

$2 = a$

So $y = 2\sqrt{x - 2.125} + 1$

Find the time when the height is 5 m

$y = 2\sqrt{x - 2.125} + 1$

$5 = 2\sqrt{x - 2.125} + 1$

$4 = 2\sqrt{x - 2.125}$

$2 = \sqrt{x - 2.125}$

$4 = x - 2.125$

$6.125 = x$

Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.

Note: Students who use an appropriate method in order to determine the starting point of the square root function have shown they have a partial understanding of the problem.

Do not penalize students who rounded their final answer.